Relationships between Drag and Periodic Vortex Shedding

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A RECENT article[1] concerned with the frequency of the vortices that result from the decomposition of a laminar wake has established the relationship

$$(R)^{1/2}/S = 16.8 + 2.8(t/l)(R)^{1/2} \tag{1}$$

that is valid when the boundary layer on the obstacle can be determined from the Blasius formulas. In the preceding expression,

t = thickness of obstacle at trailing edge
l = length of obstacle
n = frequency of vortex formation
V = freestream speed
S = nl/V = a Strouhal number
R = Vl/ν = a Reynolds number

For the case of a flat plate at zero incidence, an interesting result follows from the comparison of Eq. (1) and the drag coefficient of the plate.

The drag F_x of the plate is assumed to be the sum of two terms, one of which is due to viscous forces $F_v = 2\tau_0 l$, and the other of which is due to pressure forces $F_p = (P_\infty - P_B)t$. With this notation the drag coefficient can be written:

$$C_x = \frac{F_x}{\frac{1}{2}\rho V^2(2l)} = \frac{2\tau_0 l + (P_\infty - P_B)t}{\frac{1}{2}\rho V^2(2l)} = \frac{1.328}{(R)^{1/2}} + \frac{C_p}{2}\frac{t}{l} \tag{2}$$

where

τ_0 = surface shear stress
P_∞ = stream pressure at infinity
P_B = average pressure on base of plate
C_p = base pressure coefficient

After comparing Eq. (1) with Eq. (2), it becomes evident that

$$C_x = 0.079/S \tag{3}$$

$$C_p = 0.44 \tag{4}$$

To the writer's knowledge, there are no published data that may be compared with Eq. (3), but the proportionality between the drag coefficient and the inverse of the Strouhal number is the trend that Hoerner presents for bluff bodies in Fig. 7 of Ref. (2).

A similar result may be found from a slight modification of von Karman's classical vortex street analysis. To confine the analogy to its proper limits, it should first of all be recalled that von Karman was occupied with a perfect fluid and that his vortices are completely independent of the generating body, whereas the vortices of this study are a consequence of a transition of the body's laminar boundary layer as it crosses the trailing edge.

In the notation of Fig. 1, von Karman found the following:

Drag of Obstacle

$$F_x = \frac{\Gamma \rho h}{l}(U - 2u) + \frac{\Gamma^2 \rho}{2\pi l} \tag{5}$$

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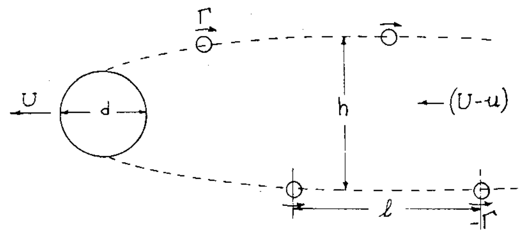

Fig. 1 Vortex street notation.

Stability Condition

$$h/l = 0.281 \tag{6}$$

Speed of Vortices

$$u = \Gamma/(8)^{1/2}l \tag{7}$$

where

ρ = fluid density
Γ = circulation of a vortex
u = speed relative to obstacle

Moreover, the vortices of either row pass a fixed point, and consequently are formed, with frequency:

$$n = (U - u)/l \tag{8}$$

or, in dimensionless form,

$$S = nl/U = 1 - u/U \tag{9}$$

The appropriate limits on S are

$$0 < S < 1 \tag{10}$$

The drag coefficient,

$$C_x = \frac{F_x}{\frac{1}{2}\rho U^2 h} = \frac{2\Gamma}{Ul}\left(1 - \frac{2u}{U}\right) + \frac{\Gamma^2}{\pi l h U^2} \tag{11}$$

in terms of the ratio u/U becomes

$$C_x = 5.64\, u/U - 2.24\,(u/U)^2 \tag{12}$$

The desired expression results from the substitution of Eq. (9) into Eq. (12):

$$C_x = 5.64(1 - S) - 2.24\,(1 - S)^2 \tag{13}$$

and is drawn in Fig. (2).

Although Eqs. (13) and (3) and Fig. 7 of Ref. (2) are not identical (very likely because they represent different problems), all express the same tendency: the drag coefficient decreases as the Strouhal number increases.

This observation is especially interesting in view of the fact that the writer's[3] experiments on flat plates alligned with the flow, and Garber's[4] on cylinders of revolution, have shown

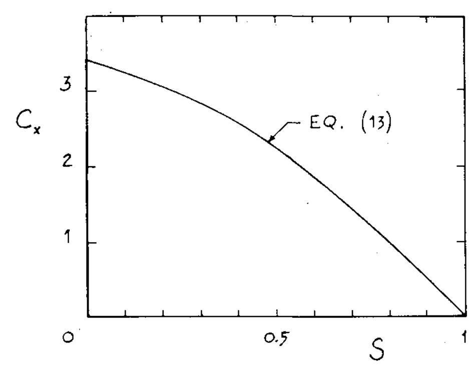

Fig. 2 Dependence of the drag coefficient C_x on the Strouhal number S for von Karman's vortex street.

that the vortex formation frequency and thus the drag coefficient, by reason of the present article, can be changed by the action of an external acoustic signal.

Additional remarks should await a thorough experimental investigation.

References

[1] Quinn, B. P., "Fréquence de la formation des tourbillons dans un sillage laminaire," Compt. Rend. Acad. Sci. Paris **258**, 5356–5358 (1964).

[2] Hoerner, S. F., *Fluid-Dynamic Drag* (published by the author, 1958), 2nd ed., Chap. III, pp. 3–6.

[3] Quinn, B. P., "Décomposition tourbillonnaire d' un sillage laminaire," Doctorate Thesis, Université de Paris (1964).

[4] Garber, D., "The effect of external sound on the vortex shedding from cylinders," J. Aeronaut. Sci. **25**, 275–276 (1958).

Dynamic Response of a Nonlinear Membrane in Supersonic Flow

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Nomenclature

a = membrane length
E = modulus of elasticity
h = membrane thickness
M = Mach number
n = number of grid points
q = dynamic pressure, $\rho U^2/2$
t = time
T = membrane tension
T_0 = membrane pretension
U = freestream velocity
V_0 = initial velocity of membrane
w = membrane deflection
w_0 = initial displacement of membrane
x = coordinate parallel to airflow
β = $(M^2 - 1)^{1/2}$
γ = membrane mass per unit area
λ = dynamic pressure parameter, $2qa/\beta T_0$
ρ = mass density of air
ω = circular frequency

Introduction

SEVERAL investigators in the field of panel flutter have given cursory attention to a degenerate case of a panel (the membrane) and have concluded that, based on linear theory, a membrane is stable when subjected to supersonic airflow over one surface. On the other hand, experimentalists have noted that panels that closely approximate membranes appear to flutter physically.

In view of this apparent experimental contradiction of the analytical prediction, there is some question concerning the effect of nonlinearities due to large deflections on the dynamic behavior of a membrane in a gas flow. The purpose of this note is to consider the nonlinear problem as an initial value problem in order to determine the effect of the nonlinearities.

Analysis

Statement of problem

An infinitely wide membrane of length a in the flow direction (x) is considered (see Fig. 1). When linearized static aerodynamic strip theory is utilized, as in Ref. 1, the nonlinear differential equation, together with the appropriate boundary and initial conditions, are

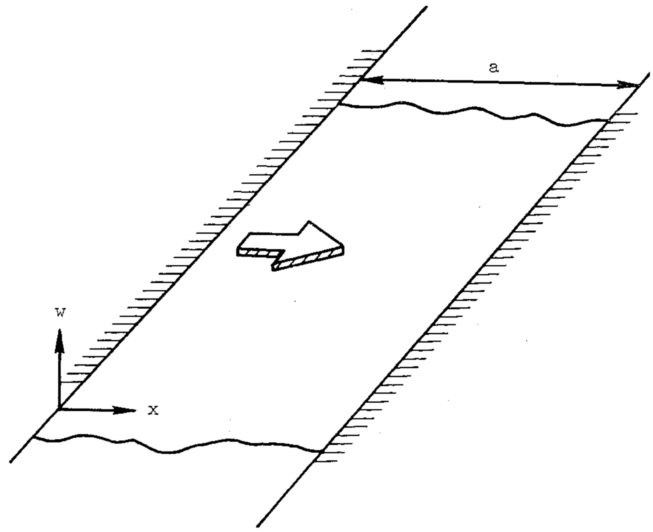

Fig. 1 Membrane geometry and coordinate system.

$$w_{xx} - \frac{2q}{\beta T} w_x - \frac{\gamma}{T} w_{tt} = 0 \qquad \left.\begin{aligned} w(0,t) &= w(a,t) = 0 \\ w(x,0) &= w_0(x) \\ w_t(x,0) &= V_0(x) \end{aligned}\right\} \tag{1}$$

where

$$T = T_0 + \frac{Eh}{2a}\int_0^a (w_x)^2\, dx \tag{2}$$

Equation (2) expresses the fact that the tension in the membrane is a function of the deflected shape and is independent of x since inplane inertia is neglected.

Solution

Linear case, exact solution. For the linear case an exact solution is

$$w = \sum_{m=1}^{\infty} A_m e^{\lambda x/2a} \sin\frac{m\pi x}{a} \cos\omega_m t \tag{3}$$

where

$$\omega_m{}^2 = \frac{T_0}{a^2\gamma}\left[\left(\frac{\lambda}{2}\right)^2 + (m\pi)^2\right] \tag{4}$$

$$A_m = 2\int_0^a e^{-(\lambda x/2a)}\, w_0 \sin\frac{m\pi x}{a}\, dx \tag{5}$$

and $V_0 = 0$.

The series (3) is seen to converge providing the series for the case $\lambda = 0$ converges. There are no values of λ which will cause an infinite displacement from a finite input; thus, for the linear case, there can be no critical value of the dynamic pressure parameter from the classical point of view, since there are no values of λ which will make ω_m [Eq. (4)] complex.

Finite-difference solution. In order to investigate the effects of the nonlinear stiffness of the membrane, a numerical procedure was used. Both the space and time derivatives in Eqs. (1) and (2) were approximated at a discrete number of points and times, respectively. The resulting set of equations was then integrated numerically to provide a time history of the response.

The total membrane energy, i.e., the sum of the kinetic and potential energies of the membrane, was used as a criterion of stability. The system was assumed to be stable if the response of the membrane energy with time were bounded, and unstable if the converse were true. Prior investigation by

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